

## Crown Volume Estimates

This paper presents a simplified method for accurately estimating the crown volumes of trees using a limited number of measurements. The thickness of the crown and the average crown spread will be measured and the general Crown form factor (Cf) of a tree will be determined by visual comparison with a chart. The volume of the crown will then be calculated by the formula:

$$\text{Crown volume} = (\text{Cf}) \times (\text{crown thickness}) \times (\text{average maximum crown spread})^2$$

Different trees have different general crown shapes. These tend to range from roughly conical, to spherical, to cylindrical. It is a reasonable assumption that trees with these differing crown shapes, even if of similar overall crown thickness and spread would have different volumes, and that these volumes would progressively vary as the overall crown shape varied from one form to another.

### CONCEPT

The crown of a tree is a three dimensional object that may be thought of as its visual profile rotated about the trunk of the tree. This solid form can be modeled as a series of disks stacked one atop another of varying diameters, each diameter equal to the average crown diameter at that height. The more disks, the closer this disk stack will approximate the volume of the crown. This is one of the basic principles of calculus. Volumes of the crown of a tree can be measured therefore, by climbing the tree and measuring the different diameters of the crown at different heights, calculating the volume of each of these disks, and totaling them together. The limbs are not exactly the same length in each direction on a tree, but an average length can be used for calculating the volume each individual disk. Consider that there must be a single cylinder of the same height as the crown thickness that has the same volume as the irregularly shaped crown. The problem then becomes one of determining the diameter of this cylinder so that its volume equals that of the crown of the tree. The volume of each of the individual disks can be calculated by using the formula for the volume of a cylinder:

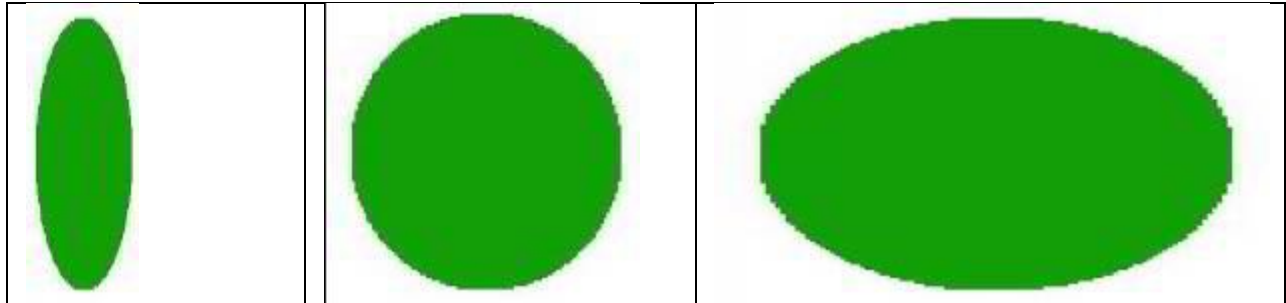
$$\text{Volume of disk} = (\text{pi})(\text{height})(\text{radius}^2) = (\text{pi})(\text{height})(\text{diameter}^2)/4$$

By rearranging the numbers you can derive a formula for the radius needed for the single cylinder solution. The height and pi drop out and the result is the needed radius is equal to the square root of the average of the radius<sup>2</sup> for each of the disks.

$$\text{radius}_{(\text{cylinder})} = [\text{AVERAGE} (r_1^2 + r_2^2 + \dots + r_x^2)]^{0.5}$$

The key to understanding this is that the absolute length of each radius is important, but also how they change in length relative to each other at different heights is just as important. This progression of relative lengths may be thought of as the form of the crown. For any given form

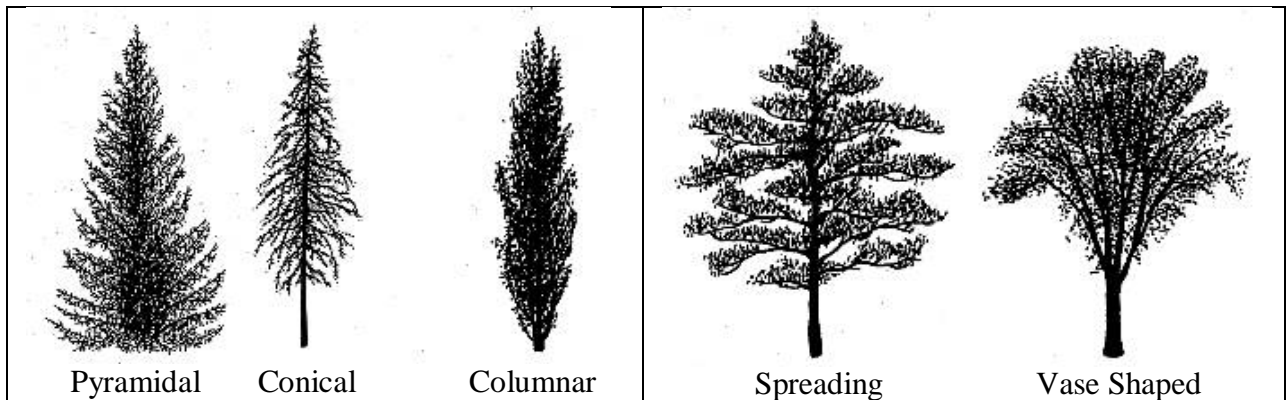
of the crown, the length of this single cylinder radius and single cylinder diameter will be proportional to the measured average maximum crown spread of the tree.

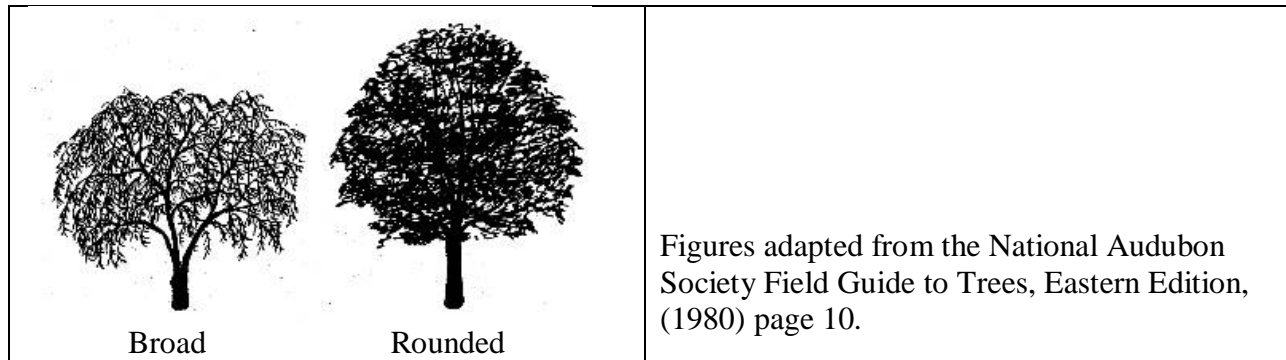


Each of these shapes above represent idealized tree crowns that would have the same crown form, meaning that in each of these shapes the spread varies proportionally the same way at different heights in the crown. The ration of the simple cylinder diameter to average crown would be the same in each example.

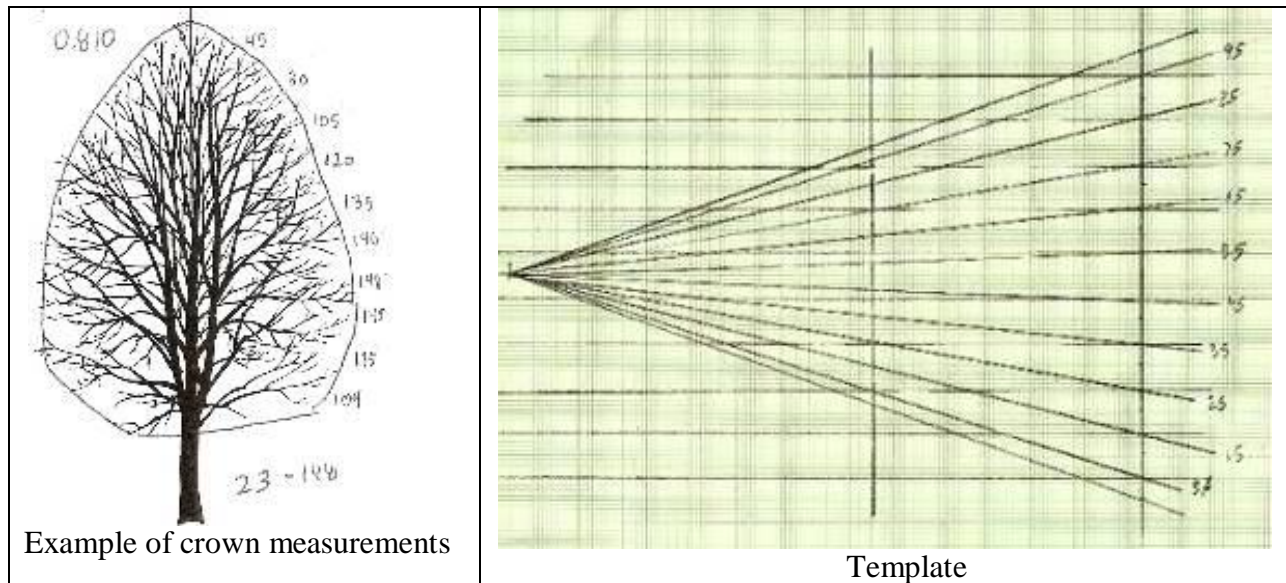
#### CALCULATION OF IDEALIZED CROWN VOLUMES

The next step is to calculate the volumes of a variety of crowns of different forms. Calculus can be used to calculate the volume of any shape rotated about an axis; however there is no series of equations that define various crown shapes. Therefore a graphical solution was employed. The National Audubon Society Field Guide to Trees, Eastern Edition, (1980) page 10, lists seven tree shapes:





This is a reasonable classification of general tree form, however, there were not sufficient numbers of examples to complete adequate calculations of canopy volumes for each form. Peterson Field Guides Eastern Trees, by George A. Petrides and illustrated by Janet Wehr, (1998) has a chapter on Tree silhouettes by Roger Troy Peterson. In it are illustrations of 48 different tree silhouettes. These were used as a basis to make crown volume calculations. It is not relevant whether or not these are idealized forms or illustrations representing true examples. What is important is that there was a broad variety of shapes of differing forms presented that could be measured. Photocopies of these drawing were annotated. First the crown of the tree was outlined and a centerline was drawn vertically through the illustration marking the center point of the tree.



The tree was divided into ten equal height vertical segments, and the center point of each of these segments was determined with a variable template. The width of the crown in the illustration was measured at 5%, 15%, 25%, 35%, 45%, 55%, 65%, 75%, 85%, and 95% of the height of the crown. These values represent the average of the diameter of each of the disks making up

volume of the crown. Then using these values, and a variation of the formula presented above the diameter of a single cylinder of equal volume to the crown of the tree was calculated. Of the 48 illustrations, 44 were used in the measurement process. Those not used included one multitrunk example, and three examples of smaller trees/shrubs that were too asymmetrical to provide useful comparisons. I included a drawing of a clump of pussy willows for comparisons.

## STANDARD GEOMETRIC FORMS

Similar calculations can be made for several standard geometrical forms that are similar to tree canopy shapes.

$$\text{Volume of a cylinder} = (\pi) (h) (r^2)$$

$$\text{The ratio of average crown spread/diameter of cylinder} = 1$$

$$\text{Volume of a Sphere} = (4/3) (\pi) (r^3)$$

$$\text{The ratio of average radius of a sphere/ diameter of cylinder} = 0.8165$$

$$\text{Volume of a cone} = (1/3) (\pi) (r^2)$$

$$\text{The ratio of average radius of a cone/ diameter of cylinder} = 0.577$$

## RATIO OF AVERAGE CROWN DIAMETER TO AVERAGE MAXIMUM SPREAD

The results generally are what would be expected. Those trees with a more conical shape are at the bottom end of the range, while those trees with some almost cylindrical segments are in the higher range. The results are presented on the table below. The tree species listed are those used by Peterson to denote the respective silhouettes. The example with the lowest ratio was the illustration of the white spruce illustration with a 0.679. This is still substantially higher than that of a simple cylinder. The example with the highest ratio was eastern sycamore illustration with a ratio of 0.897. The numerical average of the entire set was a ratio of 0.800. It is surprising that the variation between the maximum and minimum ratio is so small. The range of the entire measured set fell between -12.1% and +9.7% of the average value for the set in spite of the dramatic variations of overall shape.

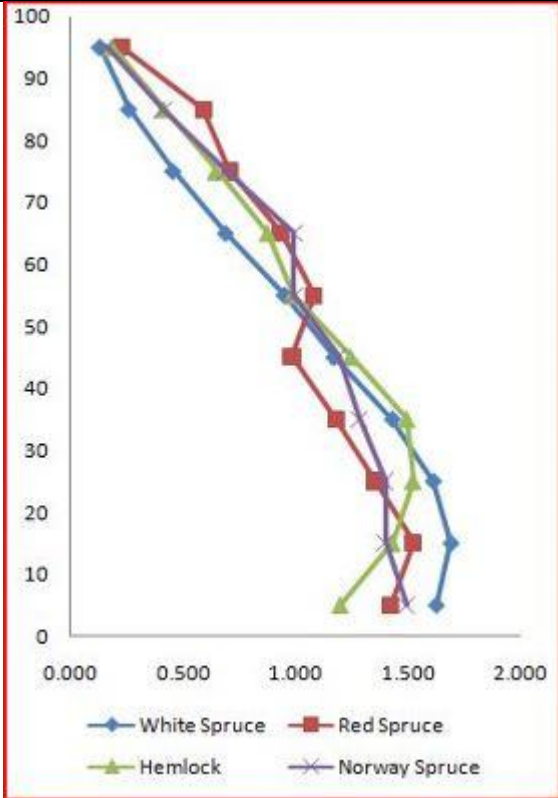
Some general observations can be made. Those trees having a pyramidal to conical shape fell in the range of 0.679 to 0.729. The next category could be described as spade shaped with a rounded base section and a triangular shaped point. These fell in the ratio range from 0.753 to 0.785. The next group had a range of shapes from more elongated spades, to round, to oval and the ratio ranged from 0.804 to 0.836. The final group were spreading, generally broad crowned

trees that tended to have vertical segments of their crown represented by longer limbs all of similar length, essentially vertical sides in sections. These ratios ranged from 0.847 to 0.897. There were only three examples of vase-shaped or upswept trees. Two of them respectively had ratios of 0.762 and 0.772. This seems an appropriate range for this form. The other, an elm, had a ratio of 0.835, but while this tree had upswept limbs, the crown could better be described as round in shape. The Audubon Guide listed a category of columnar but this referred to the fact that the limbs of these trees were short relative to the tree height. In terms of form they generally were better categorized as pyramidal to spade shaped with ratios between 0.685 and 0.781. It is important when applying these criteria to analyze branch length pattern rather than branch length itself.

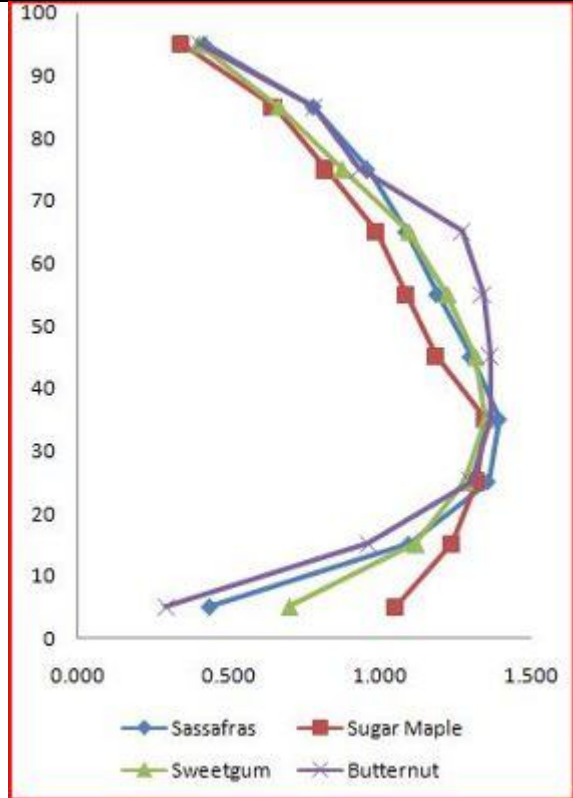
| Species            | Ratio | Species          | Ratio |
|--------------------|-------|------------------|-------|
| Cone               | 0.577 | Honey Locust     | 0.816 |
| White Spruce       | 0.679 | Sphere           | 0.817 |
| Northern Red Cedar | 0.685 | White Oak        | 0.817 |
| Red Spruce         | 0.704 | White Oak        | 0.817 |
| Balsam Fir         | 0.715 | Tuliptree        | 0.824 |
| Hemlock            | 0.718 | Bur Oak          | 0.826 |
| Eastern Red Cedar  | 0.722 | Pitch Pine       | 0.829 |
| Norway Spruce      | 0.729 | American Elm     | 0.830 |
| Sassafras          | 0.753 | Loblolly Pine    | 0.835 |
| Bald Cypress       | 0.762 | American Elm     | 0.835 |
| Lombardy Poplar    | 0.765 | Red Maple        | 0.835 |
| Sugar Maple        | 0.766 | Black Cherry     | 0.836 |
| Eastern Cottonwood | 0.772 | Pignut Hickory   | 0.848 |
| Black Willow       | 0.776 | Dogwood          | 0.849 |
| Sweetgum           | 0.776 | Weeping Willow   | 0.849 |
| Sugar Maple        | 0.777 | White Pine       | 0.850 |
| Quaking Aspen      | 0.777 | Shagbark Hickory | 0.850 |
| Black Spruce       | 0.782 | Osage-orange     | 0.854 |
| Tamarack           | 0.785 | Beech            | 0.868 |
| Butternut          | 0.785 | Catalpa          | 0.882 |
| Pussy Willow clump | 0.804 | Black Locust     | 0.889 |
| White Ash          | 0.810 | Eastern Sycamore | 0.897 |
| Red Pine           | 0.812 | Average          | 0.800 |
| Pin Oak            | 0.815 |                  |       |

## NORMALIZED CROWN SHAPE

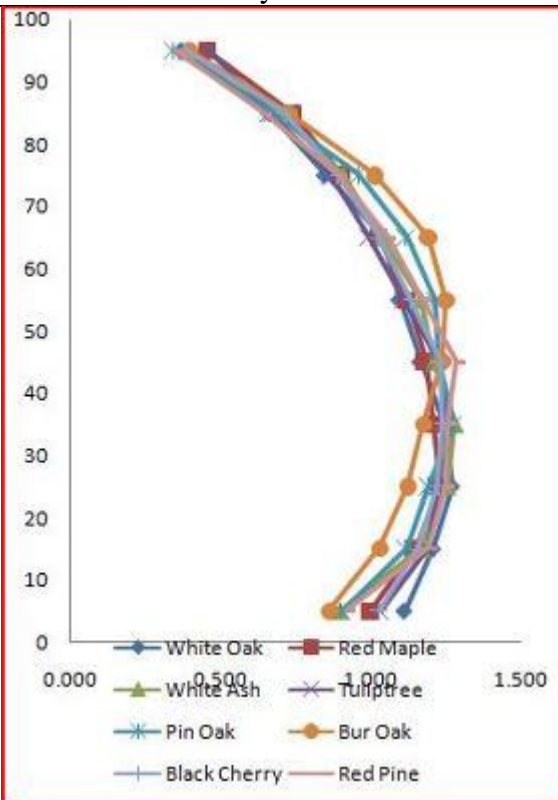
For each diameter of each crown profile measured, a normalized crown diameter/limb length was calculated to determine the limb length pattern and the overall shape of the tree. The measured diameters were divided by the average diameter to normalize these values.



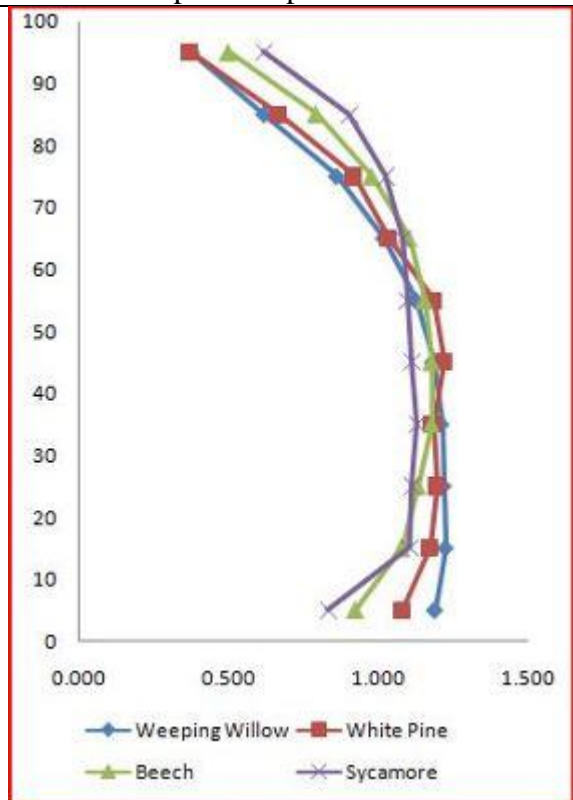
Conical to Pyramidal Forms



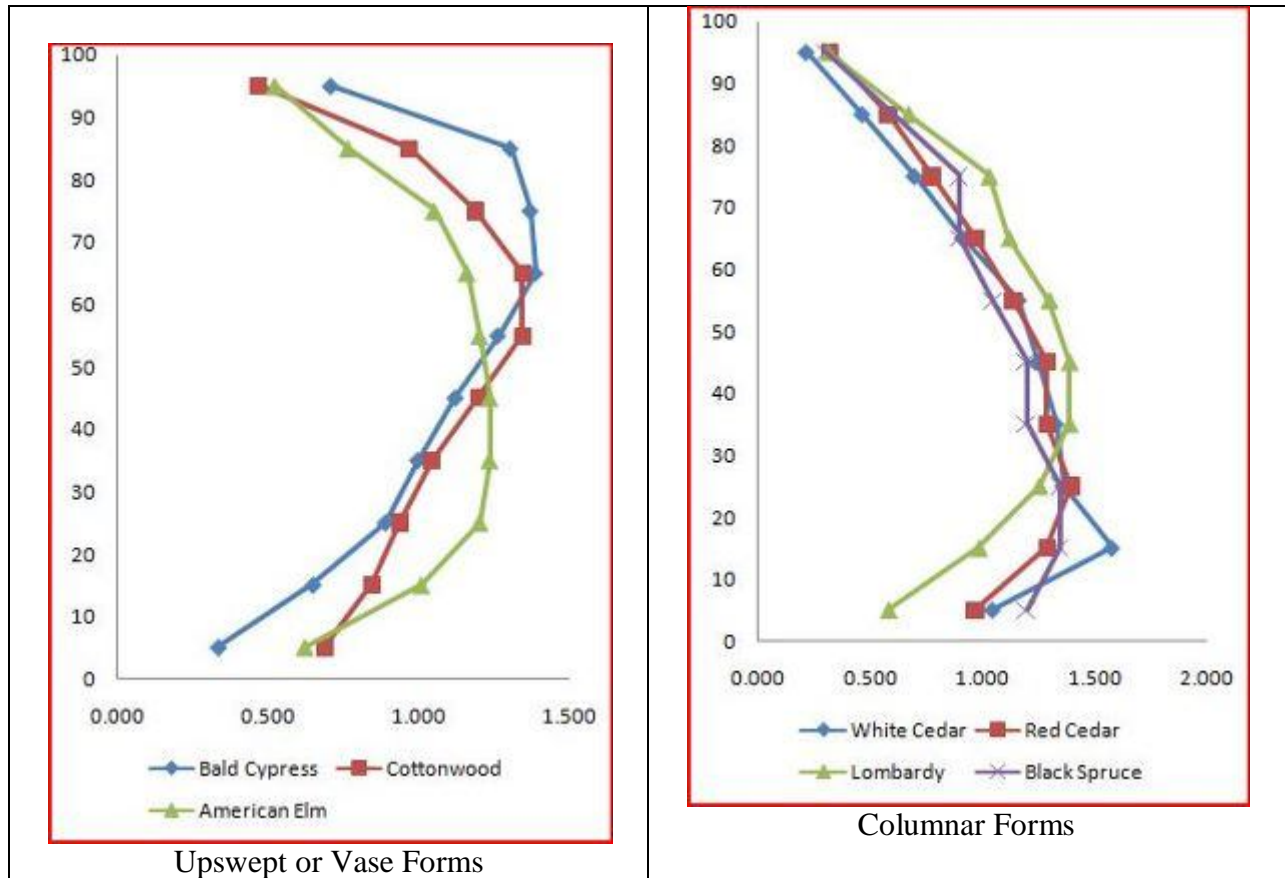
Spade-shaped Forms



Rounded Forms



Spreading to Cylindrical Forms



Normalized shape scatter for select trees.

## CROWN FORM FACTOR

The ratios of the measured maximum average crown spread to radius of the equivalent cylinder diameter will not be used directly but first must be converted to a crown form factor value.

The formula for an equivalent cylinder may be expressed as follows:

$$\text{Volume cylinder} = \pi \times h \times (\text{ratio} \times r)^2 = [\pi \times h \times (\text{ratio} \times \text{average crown spread})^2] / 4,$$

where average maximum crown spread = 2 average maximum radius

All of the constants can be converted to a single crown form factor:

$$Cf = [\pi \times (\text{ratio})^2] / 4$$

And this can be applied to the initial formula presented:

$$\text{Crown volume} = (Cf) \times (\text{crown thickness}) \times (\text{average maximum crown spread})^2$$

## SPECIAL CASES

There are a couple of special cases that need consideration. The first is the case of a domed shaped canopy, such as found in a number of live oak trees that can better be modeled as the top section of a hemisphere. A tree crown fits this shape model if: a) it has a domed shaped top surface, b) the base of the crown is flat or at ground level on a flat surface, and 3) the width of the crown spread is greater than or equal to twice the vertical thickness of the crown. Because of the shape variations this form can best be numerically evaluated. Robert Leverett developed an Excel spreadsheet that automatically calculates the volume of this section given the crown height and average maximum crown spread and submitted it to the ENTS discussion list on February 24, 2009.

The second special case is where the crown of the tree is exceptionally asymmetrical. In most cases averaging the length of the maximum and minimum axis of the crown will produce an acceptable result. In extreme cases each horizontal axis can be entered separately into this formula:

$$\text{Crown volume} = (Cf) \times (\text{crown thickness}) \times (\text{maximum axis}) \times (\text{minimum axis})$$

This formula includes the hidden assumption that the shape of the crown is similar perpendicular to both axis.

## IMPLEMENTATION OF METHODOLOGY

The thickness of the crown and the average crown spread will be measured and the general Crown form factor (Cf) of a tree will determined by visual comparison with a chart. It must be noted that this methodology does not calculate crown density or health.

Using the ENTS criteria average crown spread is obtained by measuring the longest and shortest extent of the crown and averaging the figures. Alternatively it may be measured by averaging the length of a series of maximum limb radii measured from four or more positions spaced around the tree and multiplying the result by two.

The crown height is the vertical distance between the base of the green crown and the top of the crown ignoring any epicormic sprouts or suckers and any stray sprigs that may overtop the general mass of the crown of the tree.

The chart below provides graphic examples of different crown forms. The user can compare the forms illustrated with those of the tree being examined to find the best match. It is important that the pattern of change in branch length be examined rather than the actual length of the branches when making this determination of which forms best match. Extraneous branches and sprigs that

make up a small portion of the volume of the crown and that extend beyond the general mass of the crown itself should be ignored, as should hollows within the mass of the crown.

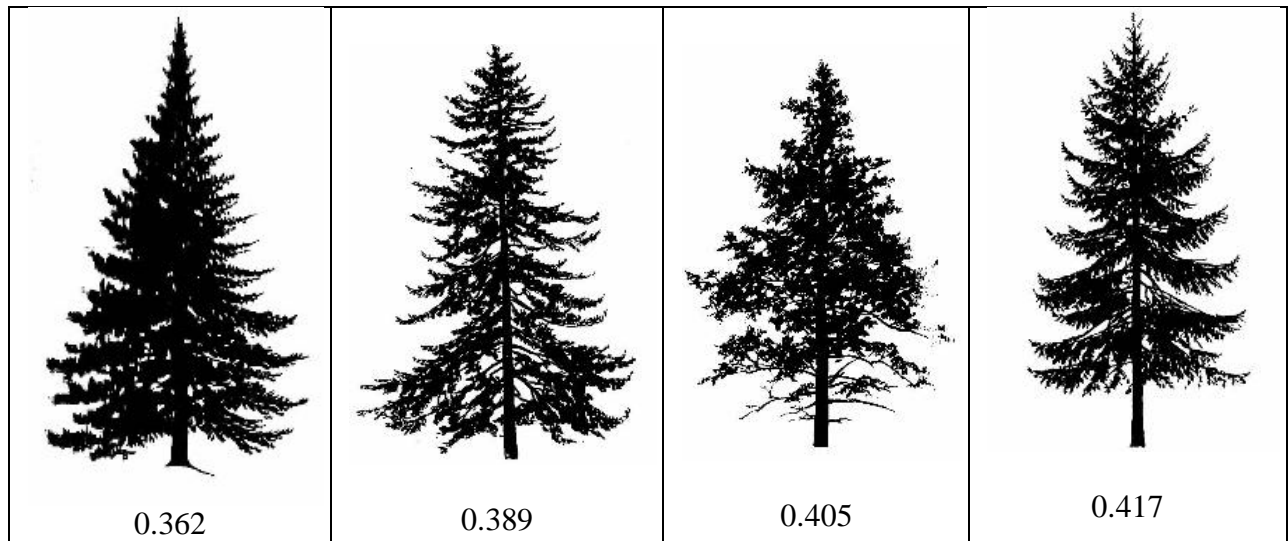
Each tree crown form on the chart is accompanied by a Cf value for that particular shape. These three values are then used in the formula presented above to determine crown volume for the tree.

In cases of trees with unusually shaped crowns, if a photograph of the crown can be taken from a distance to mitigate distortion the methodology described above for calculating idealized crown volumes can be applied to these trees to derive the Cf. With measurements of average maximum crown spread and crown thickness and this individualized Cf the volume of this individual crown can be calculated.

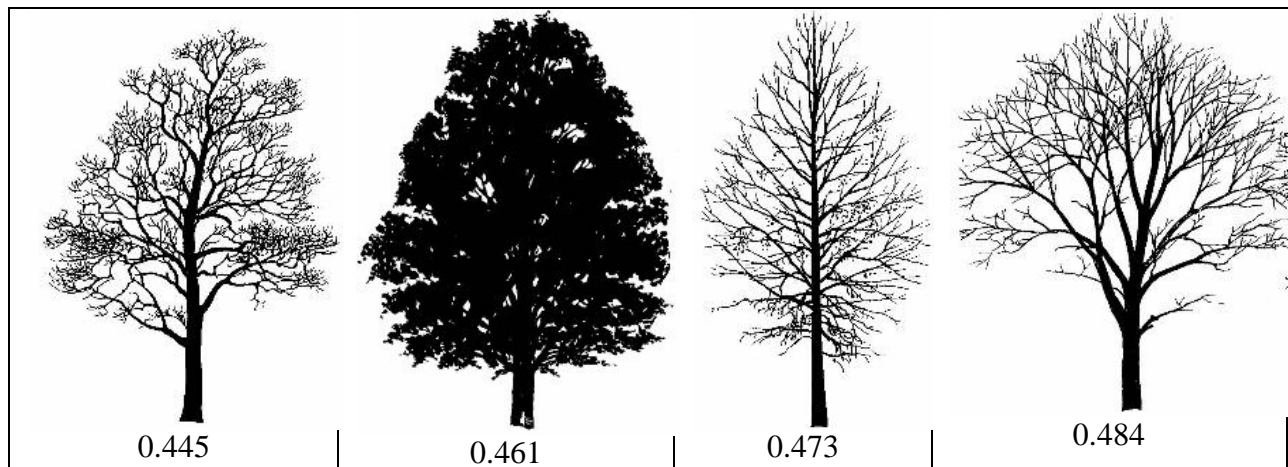
Edward Frank

February 27, 2009

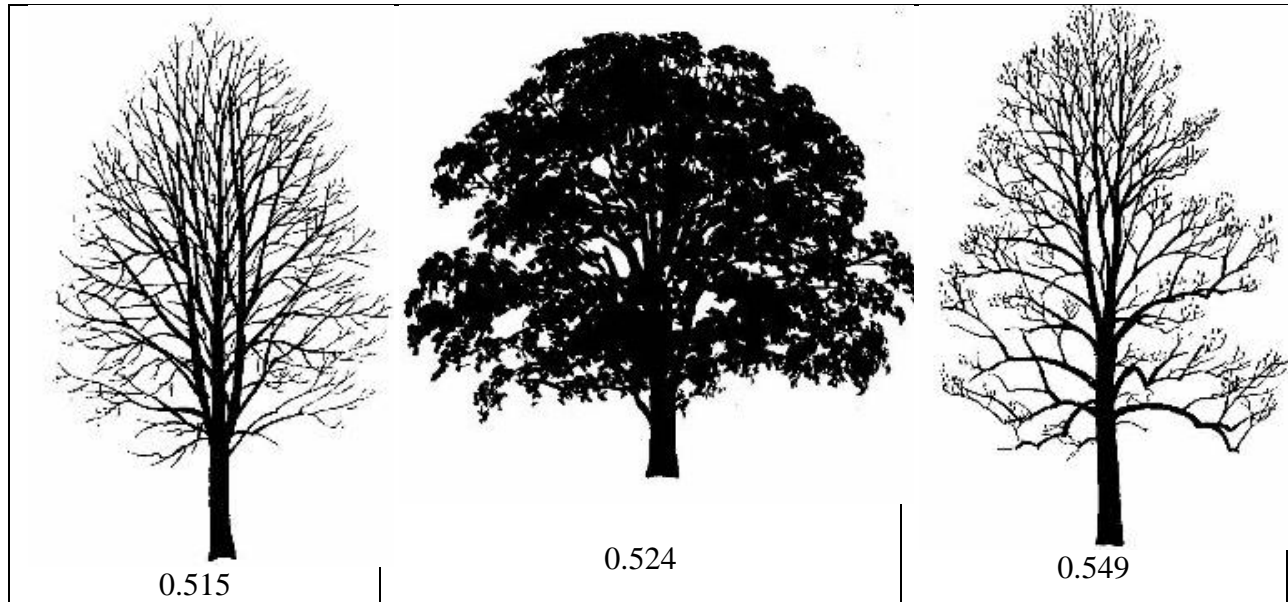
Conical to Pyramidal Forms: Cf values range from 0.362 to 0.417



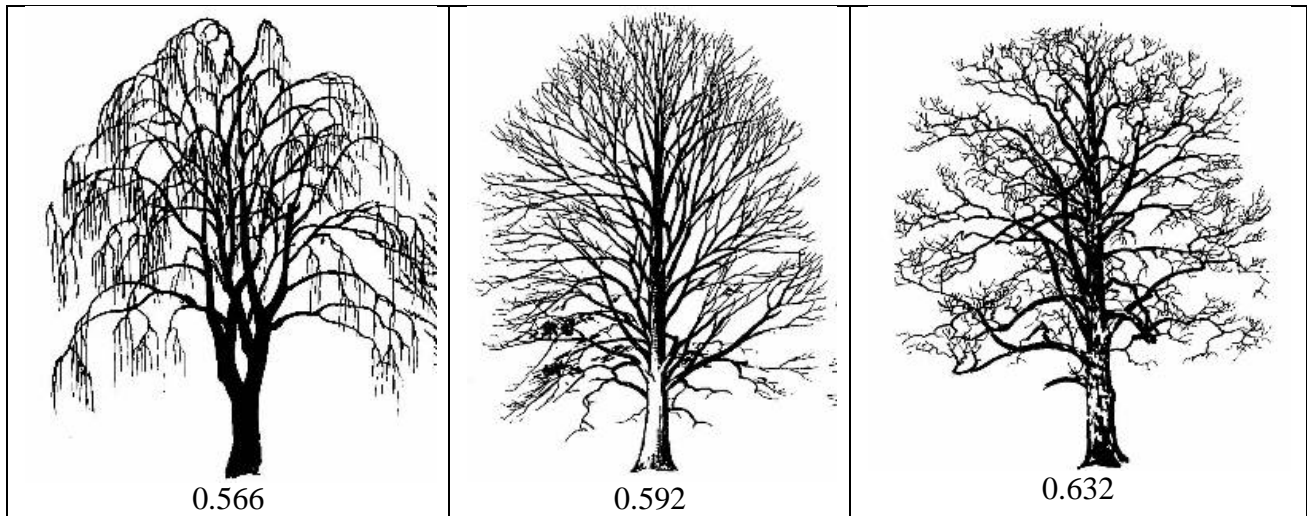
Spade Shaped Forms: Cf values range from 0.445 to 0.484



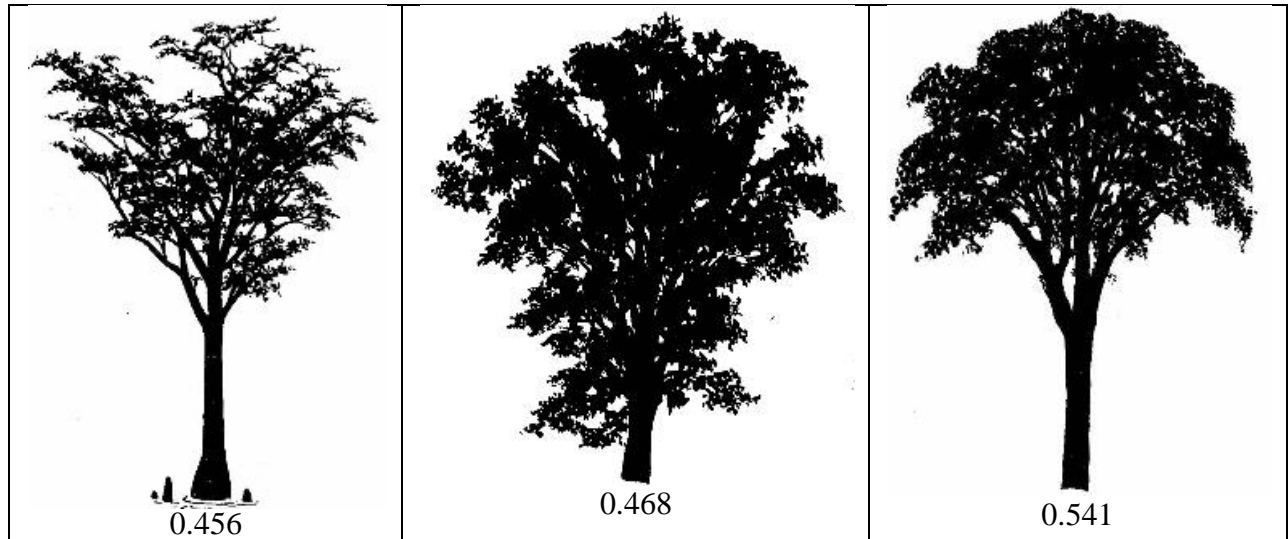
Elongate Spade to Rounded to Oval Shapes: Cf values range from 0.508 to 0.549



Spreading to Cylindrical Forms: Cf values range from 0.565 to 0.632



Upswept and Vase Shapes: Cf ranges from 0.456 to 0.468. The American Elm in the examples has an upswept branch form but the crown itself is rounded in form.



Columnar Forms:

