# Crown Volume Estimates 

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## Introduction

This paper presents a simplified method for estimating the crown volumes of trees using a limited number of measurements. Looking at a tree standing in a field a profile of the crown can be seen. Different trees have different general crown profiles and crown shapes. The overall shape of the crown will tend to range from roughly conical, to spherical, to cylindrical. It is a reasonable assumption that trees with these differing crown shapes, even if of similar overall crown thickness and spread would have different volumes, and that these volumes would progressively vary as the overall crown shape varied from one form to another.

The volume of the crown can be determined using three values: 1) crown spread, 2) crown thickness, and 3) crown shape. The thickness of the crown and the average crown spread will be measured and the general crown shape of the tree will determined by visual comparison with a chart. The crown shape will be used to derive a Crown Form (CF) value for different tree shapes and will be the third parameter of the crown volume calculation formula.

Crown spread can be measured in several different ways. Commonly a diameter of the crown is measured along its widest point and a second measurement is taken at right angles to the first measuring the width of the crown in this second axis. The two are averaged to determine the average crown spread. Alternatively a series of spokes are measured from the outer extension of the branches to the center of the tree. These are averaged and multiplied by two to calculate an average crown spread. In reality this is actually the average of the maximum spread in each axis or spoke measured and should be thought of as the average maximum crown spread.

Crown thickness is simply the difference in height from the base of the crown, ignoring stray branches and epicormic sprouts, and the top of the crown or top of the tree. This parameter is measured using standard height measurement techniques. Crown spread measurements and height measurements use the standard ENTS methodology presented by Blozan (2004)

Crown shape is the third parameter. The crown of a tree is a three dimensional object that may be thought of as this visual profile rotated about the trunk of the tree.


Figure 1: Crown profile rotation

One method of determining the volumes of the crown of a tree is by climbing the tree and doing detailed mapping of each portion of the crown and adding these mapped volumes together. This is a very time consuming process and may not be practical in many situations.

In the methodology presented here the profile of a tree is compared visually to a set of standards and a best match is selected. Each different profile has an associated CF value that can then be used determine crown volume.

## Discussion

A solid form can be modeled as a series of disks stacked one atop another of varying diameters, each diameter equal to the average crown diameter at that height. The more disks, the closer this disk stack will approximate the volume of the crown. This is one of the basic principles of calculus. Different diameters of the crown can be measured at different heights, calculating the volume of each of these disks, and totaling them together. The limbs are not exactly the same length in each direction on a tree, but an average length can be used for calculating the volume each individual disk.

Consider that there must be a single cylinder of the same height as the crown thickness that has the same volume as the irregularly shaped crown. The problem then becomes one of determining the diameter of this cylinder so that its volume equals that of the crown of the tree. The volume of each of the individual disks can be calculated by using the formula for the volume of a cylinder:

Volume of disk $=(\pi)($ height $)\left(\right.$ radius $\left.^{2}\right)=(\pi)($ height $)\left(\right.$ diameter $\left.^{2}\right) / 4$

By rearranging the numbers you can derive a formula for the radius needed for the single cylinder solution. The height and $\pi$ drop out and the result is the needed radius is equal to the square root of the average of the radius ${ }^{2}$ for each of the disks.

$$
\operatorname{radius}_{(\mathrm{cylinder})}=\left[\text { AVERAGE }\left(\mathrm{r}_{1}^{2}+\mathrm{r}_{2}^{2}+\ldots+\mathrm{r}_{\mathrm{x}}^{2}\right]^{0.5}\right.
$$

The key to understanding this is that the absolute length of each radius is important, but also how they change in length relative to each other at different heights is just as important. This progression of relative lengths may be thought of as the form of the crown. For any given Crown Form, the length of this single cylinder radius and single cylinder diameter will be proportional to the measured average maximum crown spread of the tree.


Figure 2: Different idealized crown shapes with the same Crown Form value.

Each of these shapes above represent idealized tree crowns and in each of these shapes the spread varies proportionally the same way at different heights within the crown. Therefore they have the same overall Crown Form. All that varies among the shapes is the overall diameter of the crown. The ratio of the simple cylinder diameter to average crown would be the same in each example.

## Evaluation of Idealized Crown Forms

The next step is to calculate the volumes of a variety of crowns of different forms. Calculus can be used to calculate the volume of any shape rotated about an axis; however there is no series of equations that define various crown shapes. Therefore a graphical solution was employed. The National Audubon Society Field Guide to Trees, Eastern Edition, (1980) page 10, lists seven tree shapes:


Figure 3: Tree shape classification as adapted from the National Audubon Society Field Guide to Trees, Eastern Edition, (1980) page 10.

This is a reasonable classification of general tree forms, however, there were only a limited numbers of tree form examples presented in the text. Peterson Field Guides Eastern Trees, by

George A. Petrides and illustrated by Janet Wehr, (1998) has a chapter on Tree silhouettes by Roger Troy Peterson. In it are illustrations of 48 different tree silhouettes. These illustrations of idealized tree forms were used as a basis to make crown volume calculations. For purposes of this type of analysis it does not matter if the idealized form shown for tuliptrees was actually representative of all tuliptrees or not. What is important is that there was a wide d variety of tree profiles representing the overall continuum of actual tree shapes presented that could be measured. Photocopies of these drawing were annotated. First the crown of the tree was outlined and a centerline was drawn vertically through the illustration marking the center point of the tree.


Figure 4: Illustration of the spread measurement process.

The tree was divided into ten equal height vertical segments, and the center point of each of these segments was determined with a variable template. The width of the crown in the illustration was measured at $5 \%, 15 \%, 25 \%, 35 \%, 45 \%, 55 \%, 65 \%, 75 \%, 85 \%$, and $95 \%$ of the height of the crown. These values represent the average of the diameter of each of the disks making up volume of the crown. Then using these values, and a variation of the formula presented above
the diameter of a single cylinder of equal volume to the crown of the tree was calculated. Of the 48 illustrations, 44 were used in the measurement process. Those not used included one multitrunk example, and three examples of smaller trees/shrubs that were too asymmetrical to provide useful comparisons. I included a drawing of a clump of pussy willows for comparisons.

## Standard Geometric Forms

Similar calculations can be made for several standard geometrical forms that are similar to tree canopy shapes.

Volume of a cylinder $=(\pi)(h)\left(\mathrm{r}^{2}\right)$
The ratio of average crown spread/diameter of cylinder $=1$
Volume of a Sphere $=(4 / 3)(\pi)\left(r^{3}\right)$
The ratio of average radius of a sphere/ diameter of cylinder $=0.8165$
Volume of a cone $=(1 / 3)(\pi)\left(r^{2}\right)$
The ratio of average radius of a cone/ diameter of cylinder $=0.577$

## Ratio of Average Crown Diameter to Average Maximum Crown Spread

The results generally are what would be expected. Those trees with a more conical shape are at the bottom end of the range, while those trees with some almost cylindrical segments are in the higher range. The results are presented on the table below. The tree species listed are those used by Peterson to denote the respective silhouettes. The example with the lowest ratio was the illustration of the white spruce illustration with a 0.679 . This is still substantially higher than that of a simple cylinder. The example with the highest ratio was eastern sycamore illustration with a ratio of 0.897 . The numerical average of the entire set was a ratio of 0.800 . It is surprising that the variation between the maximum and minimum ratio is so small. The range of the entire measured set fell between $-12.1 \%$ and $+9.7 \%$ of the average value for the set in spite of the dramatic variations of overall shape.

Some general observations can be made. Those trees having a pyramidal to conical shape fell in the range of 0.679 to 0.729 . The next category could be described as spade shaped with a rounded base section and a triangular shaped point. These fell in the ratio range from 0.753 to 0.785. The next group had a range of shapes from more elongated spades, to round, to oval and the ratio ranged from 0,804 to 0.836 . The final group were spreading, generally broad crowned trees that tended to have vertical segments of their crown represented by longer limbs all of
similar length, essentially vertical sides in sections. These ratios ranged from 0.847 to 0.897 . There were only three examples of vase-shaped or upswept trees. Two of them respectively had ratios of 0.762 and 0.772 . This seems an appropriate range for this form. The other, an elm, had a ratio of 0.835 , but while this tree had upswept limbs, the crown could better be described as round in shape. The Audubon Guide listed a category of columnar but this referred to the fact that the limbs of these trees were short relative to the tree height. In terms of form they generally were better categorized as pyramidal to spade shaped with ratios between 0.685 and 0.781 . It is important when applying these criteria to analyze branch length pattern rather than branch length itself.

The names assigned by the Petersons' Guide have been used in the tabulations and graphs of the results for illustrative purposes. This does not imply that the form of all trees of a particular species will have the same overall profile and will have the same Crown Form.

| Species | Ratio | Species | Ratio |
| :--- | :--- | :--- | :--- |
| Cone | 0.577 | Honey Locust | 0.816 |
| White Spruce | 0.679 | Sphere | 0.817 |
| Northern Red Cedar | 0.685 | White Oak | 0.817 |
| Red Spruce | 0.704 | White Oak | 0.817 |
| Balsam Fir | 0.715 | Tuliptree | 0.824 |
| Hemlock | 0.718 | Bur Oak | 0.826 |
| Eastern Red Cedar | 0.722 | Pitch Pine | 0.829 |
| Norway Spruce | 0.729 | American Elm | 0.830 |
| Sassafras | 0.753 | Loblolly Pine | 0.835 |
| Bald Cypress | 0.762 | American Elm | 0.835 |
| Lombardy Poplar | 0.765 | Red Maple | 0.835 |
| Sugar Maple | 0.766 | Black Cherry | 0.836 |
| Eastern Cottomwood | 0.772 | Pignut Hickory | 0.848 |
| Black Willow | 0.776 | Dogwood | 0.849 |
| Sweetgum | 0.776 | Weeping Willow | 0.849 |
| Sugar Maple | 0.777 | White Pine | 0.850 |
| Quaking Aspen | 0.777 | Shagbark Hickory | 0.850 |
| Black Spruce | 0.782 | Osage-orange | 0.854 |
| Tamarack | 0.785 | Beech | 0.868 |
| Butternut | 0.785 | Catalpa | 0.882 |
| Pussy willow clump | 0.804 | Black Locust | 0.889 |
| White Ash | 0.810 | Eastern Sycamore | 0.897 |
| Red Pine | 0.812 |  | 0.800 |
| Pin Oak | 0.815 |  |  |

Figure 5: Summary table of the ratio of the diameter of the equivalent cylinder volume to the average maximum crown spread for the illustration of the species.

## Normalized Crown Shapes

For each diameter of each crown profile measured, a normalized crown diameter/limb length was calculated to determine the limb length pattern and the overall shape of the tree. The measured diameters were divided by the average diameter to normalize these values. The average diameter used to normalize the values is the diameter of the cylinder calculated to be equivalent in volume to that of the crown of that particular shape. Six distinct form groups emerged from the data.


Figure 6: Conical to Pyramidal forms


Figure 7: Spade-shaped forms


Figure 8: Rounded forms


Figure 9: Spreading to Cylindrical forms


Figure 10: Upswept or Vase forms


Figure 11: Columnar forms

## Crown Form Factor

Each different crown shape will have an associated crown shape ratio of the measured maximum average crown spread to radius of the equivalent cylinder diameter. This value cannot be used directly but first must be converted to a unique Crown Form factor value.

The formula for an equivalent cylinder may be expressed as follows:
Crown volume $=$ Volume equivalent cylinder $=(\pi)\left(\mathrm{h}(\mathrm{r})^{2}=(\pi)\right.$ (thickness of crown) $\left[(\text { crown shape ratio)(average maximum crown spread) }]^{2} / 4\right.$, where average maximum crown spread $=2$ average maximum radius

The constants can be rearranged to derive a Crown Form factor:

$$
\mathrm{CF}=\left[(\pi)(\text { crown shape ratio })^{2}\right] / 4
$$

The overall volume equation can then be rewritten as follows:
Crown volume $=(\mathrm{CF}) \times($ crown thickness $) \times(\text { average maximum crown spread })^{2}$
Thus the complex problem of estimating crown volume is reduced to two easily measured parameters - average maximum crown spread and crown thickness, and one value that can be determined using visual matching of shapes from a table of standard shapes. The figures below provide graphic examples of different crown shapes. The user can compare the forms illustrated with those of the tree being examined to find the best match. It is important that the pattern of change in branch length be examined rather than the actual length of the branches when making this determination of which forms best match. Extraneous branches and sprigs that make up a small portion of the volume of the crown and that extend beyond the general mass of the crown itself should be ignored, as should hollows within the mass of the crown. Each tree crown profile on the chart is accompanied by a CF value for that particular shape.

## Results

The result of this process is the generation of a series of crown profile shapes and associated CF values that can be used in the field to determine the CF value of a particular tree.


Figure 12: Conical to Pyramidal Forms: CF values range from 0.362 to 0.417


Figure 13: Spade Shaped Forms: CF values range from 0.445 to 0.484


Figure 14: Elongate Spade to Rounded to Oval Shapes: CF values range from 0.508 to 0.549


Figure 15: Spreading to Cylindrical Forms: CF values range from 0.565 to 0.632

0.456



Figure 16: Upswept and Vase Shapes: CF ranges from 0.456 to 0.468 . The American Elm in the Pederson's Guide has an upswept branch form but the crown itself is more rounded in form.


Figure 17: Columnar Forms: CF factors range from 0.369 to 0.480

## Additional Comments

There are a couple of special cases that need consideration. The first is the case of a domed shaped canopy, such as found in a number of open grown live oak trees. A CF factor and form matching chart has not been included for this particular shape, although they could be calculated. Trees of this form can be modeled as the top section of a hemisphere. A tree crown fits this shape model if: a) it has a domed shaped top surface, b) the base of the crown is flat or at ground level on a flat surface, and 3) the width of the crown spread is greater than or equal to twice the vertical thickness of the crown. Because of the shape variations this form can be easily numerically evaluated. Robert Leverett developed an Excel spreadsheet that automatically calculates the volume of this section given the crown height and average maximum crown spread and submitted it to the ENTS discussion list on February 24, 2009.

The second special case is where the crown of the tree is exceptionally asymmetrical. In most cases averaging the length of the maximum and minimum axis of the crown will produce an acceptable result. In extreme cases each horizontal axis can be entered separately into this formula:

$$
\text { Crown volume }=(\mathrm{CF}) \times \text { (crown thickness) } \times \text { (maximum axis) } \times \text { (minimum axis) }
$$

This formula includes the hidden assumption that the shape of the crown is similar in shape as bisected by both axis.

In cases of trees with unusually shaped crowns, if a photograph of the crown can be taken from a distance to mitigate distortion the methodology described above for calculating idealized crown volumes can be applied to these trees to derive the CF. With measurements of average maximum crown spread and crown thickness and this individualized CF the volume of this individual crown can be calculated.

Some trees simply have a crown shape that is too irregular to use this methodology to determine crown volume. These trees, if a crown volume value is required, will need to be evaluated in sections and the volume of the individual sections added together to determine crown volume.

## Bibliography

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Illustrations from the original posting of the article adapted from Pederson's Guide
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Figure 17: Columnar Forms: CF factors range from 0.369 to 0.480

| $0.369$ | $0.469$ |
| :---: | :---: |
| $0.460$ | $0.480$ |

